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IMAGE RECONSTRUCTION BASED ON COMPRESSIVE SAMPLING USING IRLS AND OMP ALGORITHM

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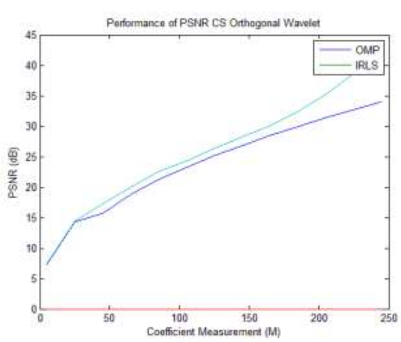
Graphical abstract

Reconstruction Image with M=50



Abstract

We proposed compressive sampling to reduce the sampling rate of the image and improve the accuracy of image reconstruction. Compressive sampling requires that the representation of the image is sparse on a certain basis. We use wavelet transformation to provide sparsity matrix basis. Meanwhile, to get a projection matrix using a random orthonormal process. The algorithm used to reconstruct the image is orthogonal matching pursuit (OMP) and Iteratively Reweighted Least Squares (IRLS). The test result indicates that a high quality image is obtained along with the number of coefficients M. IRLS has a good performance on PSNR than OMP while OMP takes the least time for reconstruction.

Keywords: compressive sampling, wavelet, random orthonormal, orthogonal matching pursuit, Iteratively Reweighted Least Squares

Abstrak

Kita mengusulkan penginderaan kompresif untuk mengurangi laju pencuplikan citra dan meningkatkan akurasi rekonstruksi citra. Penginderaan kompresif mensyaratkan bahwa representasi citra bersifat sparse pada suatu basis tertentu. Dalam penelitian ini digunakan transformasi wavelet untuk menghasilkan matrik basis sparsity. Dan menggunakan matrik proyeksi diperoleh dengan proses acak ortonormal. Algoritma rekonstruksi image menggunakan Orthogonal Matching Pursuit (OMP) dan Iteratively Reweighted Least Square (IRLS). Hasil pengujian menunjukkan bahwa gambar berkualitas tinggi diperoleh seiring dengan pertambahan jumlah koefisien M. IRLS memiliki nilai PSNR yang bagus dibandingkan OMP, sedangkan OMP membutuhkan waktu singkat pada saat rekonstruksi.

Kata kunci: penginderaan kompresif, wavelet, orthogonal matching pursuit, random orthonormal, Iteratively Reweighted Least Square

1.0 INTRODUCTION

Nowadays, digital image information is widely used in various applications such as digital cameras, remote cameras, medical images, video, and other. Images require considerable storage space and a wide bandwidth when the images transmitted. Therefore, needed a technique to reduce the amount of image data while transmitting, but the quality of images are guaranteed. A new technique in sampling and reconstructing processes were introduced, namely compressive sampling.

Compressive sampling is a method that requires signal sparsity so that the signal acquisition process in the area of transformation can be carried out under the Nyquist rate. Nyquist states that a signal to be sampled can be reconstructed into the original signal if the sampling rate is at least 2 times the largest of the frequency spectrum of the signal. While on compressive sampling, signal can be reconstructed with a lower sampling rate so that the amount of data captured within the specified time will be less than the conventional way [1]. Compressive sampling is used in many applications such as video [2], medical imaging [3] [4] [5], the seismic imaging [6], radar [7], and so on .

Compressive sampling basically had two requirements, namely the input signal sparsity and incoherency between the sparse matrix and the projection matrix [8]. Sparsity means that the information content of a signal is much smaller than the amount of data. Incoherency have a sense of uncertainty in which continuous time signal or functions may not be localized in both the time and frequency region together. The challenge is how to obtain sparse signal in certain areas so that the transformation can be done compressive sampling system in an image with a low sampling rate and high accuracy of image reconstruction.

A. Compressive sampling

Signal transformation is a process of converting signal from one domain to the other domain. Transformation from vector ***s*** to ***S*** can be expressed as the multiplication of the vector ***s*** with the transformation matrix **T** as shown in equation (1).

$$\mathbf{S} = \mathbf{Ts} \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

Wavelet is a more efficient way to approximate a natural image with a number of base elements slightly. Mathematically, the wavelet transform of the function $x(t)$ is the translation of these functions into a set of bases functions follows.

$$X_\omega(t) = \int_{-\infty}^{\infty} x(t).\,\Psi_{a,b}(t)\,dt \quad \ldots\ldots\ldots(2)$$

as $\Psi_{a,b}(t)$ is a function of the base, which is an enlarged version and a shift in the timing of the signal bandpass $\Psi(t)$ named mother wavelet and defined by the equation (3).

$$\Psi_{a,b} = \frac{1}{\sqrt{a}}\,\Psi\left(\frac{t-b}{a}\right) \ldots\ldots\ldots\ldots\ldots(3)$$

where a is the magnification parameter and b is the shift parameter.

Compressive sampling is a relatively new idea in signal processing. The main idea is to perform sampling with minimum sample where the reconstruction from the sampling results approaches the original signal. The signal has a sparse representation if it is expressed in the appropriate base.

Mathematically if a vector $x\epsilon R^n$ is represented using the wavelet basis $\Psi = [\Psi_1\Psi_2 \ldots \Psi_N]$, the vector x can be formulated [1] :

$$x = \sum_{i=1}^{N} s_{i\Psi_i}(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where s is coefficient of x that obtain from $s_i = \langle x, \Psi_1 \rangle$. Matrix form can be written as:

$$x = \Psi s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

where Ψ is N x N matrix, x and s is column vector N x 1

Signal x declared K-sparse if only K of coefficients **s** worth is not zero while (N-K) coefficient is zero. On compressive sampling takes M sample of the signal, where $K < M \ll N$.
On a linear measurement to calculate y as inner product between x and the set of vector $\{\Phi_j\}_{j=1}^{M}$ is $y_j = \langle x, \Phi_j \rangle$, can be expressed in matrix form [9]:

$$y = \Phi x = \Phi\,\psi\,s = \theta s \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

with y an output signal that has been compressed, Φ is a projection/measurement matrix sized M x N , Ψ is the base matrix sized N x N, and $\theta = \Phi\,\psi$ is reconstruction matrix sized M x N.

Compressive sampling method requires two the base matrix, which is the projection/measurement base Φ and sparse base Ψ are incoherent. Incoherent means the duality between time and frequency and expresses the idea that objects having a sparse representation in Ψ must be spread out in which they acquired, just as a Dirac or spike in the time domain is spread out in the frequency domain. The level of coherence both base determines the minimum amount of sampling to reconstruct the exact signal. Coherence illustrates the degree of similarity of the base, of little value if they are different, and valuable one if both are identical [10].

A measurement or projection is a step to determine the coefficients to be transmitted or stored. The more coefficients are selected, then the better of the reconstruction quality, but the greater the number of bits stored resulting bandwidth requirements are also getting bigger. Measurement matrix are Gaussian matrix, Bernoulli matrix, random orthogonal matrix, hadamard matrix, etc.

Sparse representation is obtained by selecting the right base. Transformation method for sparse matrix consist of discrete Fourier Transform (DCT), Discrete Wavelet Transform (DWT), Discrete Multi Wavelet Transform (DMWT), etc.

At the receiver, signal x must be recovered from the measurement results obtained, the signal y.

Classification and compressive sampling reconstruction algorithm is shown in Figure 2 [11]. Compressive sampling reconstruction algorithm are grouped into six, namely the minimization of convex, non-convex minimization, greedy algorithm, iterative thresholding, Bregman iterative, and combinatorial algorithm.

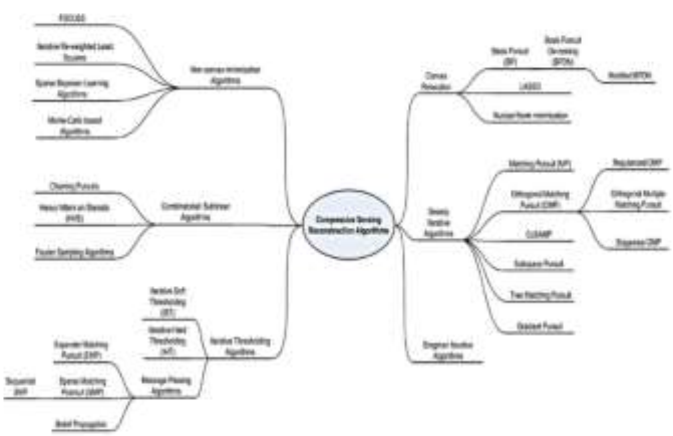


Figure 1: Compressive sampling Reconstruction Algorithm and Their Classification [11]

B. Orthogonal Matching Pursuit (OMP)

OMP is one of the group of greedy algorithms. Greedy algorithm uses an iterative approach of the coefficient signal to the signal convergence is reached, or get an approximate increase of sparse signal in each iteration by trying to calculate the measured data mismatch [12]. OMP generates a signal recovery quickly with a simple algorithm. The method starts by looking for columns that have the greatest relevance to the measurement and repeat these steps to see the correlation between the columns with the residual signal, which is obtained by subtracting the estimated contribution to the vector signal measurements.

Remember the equation (6) is a linear combination of N column from Φ. So that y has a n-term representation over dictionary Φ. Φ is a measurement matrix and denotes its column by $\Phi = \{\Phi_1, \dots \dots \dots \dots \Phi_N\}$ To identify the ideal signal x, must be determine column Φ that participation on measurement y . The idea behind the algorithm is to choose columns in greedy fashion. At each iteration, chosen column Φ which is highly correlated with the residual part of y. Then reducing any contribution to the y resulting residue. After *k* iteration, algorithm will identify the right set from the columns. The procedure of the algorithm are:

-Initialize the residual r_0, index set Δ_0, and the iteration counter $t = 1$. And Φ_o is an empty matrix.

-Find the index λ_t that solves easy optimization problem by (7). If the maximum occurs of the inner product, break the tie deterministically [13].

$$\arg mak_{j=1..M} |< r_{t-1}, \Phi_j >| \text{ ....(7)}$$

Augment the index set and the matrix of chosen atoms:

$$\Delta_t = \Delta_{t-1} \cup \{\lambda_t\} \; and \; \; \Phi_t = [\Phi_{t-1} \; \Phi_{\lambda_t}] \text{...... (8)}$$

Solve a least squares problem to obtain a new signal estimate:

$$x_t = \arg min_x \|y - \Phi_t x\|_2 \text{......................(9)}$$

Calculate the new approximation of the data and the residual:

$$a_t = \Phi_t - x_t \; and \; r_t = y - a_t \text{..............(10)}$$

Increments t and returns to equation (7) until $t < k$. The estimate x' for the ideal signal has nonzero indicates at the component listed Δ_k. The value of the estimate x' in component λ_j equal the j_{th} component of x_t.

C. Iteratively Reweighted Least Squares (IRLS)

The method of iteratively reweighted least squares (IRLS) is used to solve certain optimization problems for equation (6) with objective functions of the form $\ell_p$norm reconstruction [14], where $0 < p < 1$:

$$min \|x'\|_{p'}^{p} \text{ , subject to } \Phi x' = y \text{ ....(11)}$$

On case $p < 1$, IRLS completes (8) by replacing ℓ_p with weighted (w)ℓ_2 norm [15]:

$$min \sum_{i=1}^{N} w_i x_i'^2 \text{ , subject to } \Phi x' = y \text{ ....(12)}$$

Equation (11) can be written as:

$$min \sum_{i=1}^{N} \left| x_i'^{(n-1)} \right|^{p-2} x_i'^2 \text{ ,subject to } \Phi x' = y \text{ (13)}$$

Weight (*w*) can be calculate as first orde approximation of the objective ℓ_p:

$$w_i = \left| x_i'^{(n-1)} \right|^{p-2} \text{...............................(14)}$$

Solution of the equation (12) can be obtained from the next iteration $x'^{(n)}$:

$$x_i'^{(n)} = Q_n \Phi^T (\Phi Q_n \Phi^T)^{-1} y \text{...............(15)}$$

where Q_n as a diagonal matrix with the following value:

$$\frac{1}{w_i} = \left|x_i'^{(n-1)}\right|^{2-p} \quad \text{.............................(16)}$$

2.0 Literature Review

According to the research done by Fan Yang, et al [16], they examined image reconstruction for compressive sampling using Discrete Multi Wavelet Transform. Projection matrix is get from Bernoulli and Gaussian measurement. And reconstruction algorithm is OMP. DMWT has multi scale geometric so that provide many possible sparsity matrix. PSNR on DMWT-OMP has the best value than combination between DCT-OMP or DWT-OMP.

On the research of Sheikh Md., et al [17], Image Compression Based on CS Using Wavelet Lifting Scheme proposed the new method using sparse basis named CDf9/7 wavelet transform. Three different measurement matrix as Gaussian matrix, Bernoulli matrix, and random orthogonal matrix are used. The OMP and Basis Pursuit (BP) are applied to reconstruct each level of wavelet transform. The result showed that the CDF9/7 wavelet transform given better quality of image compressive sampling than existing method. The parameter measurement consist of PSNR/UIQI/SSIM.

3.0 Methodology

Research methodology includes quantitative as numerical processing and analyzing the data obtained from the simulation results. Part science encountered when discussing the basic theory of sampling according to Nyquist, then compared with the workings of compressive sampling.

On the theory of compressive sampling, system needed basis matrix transformation to obtain the sparse nature of signal, projection matrix, and reconstruction algorithm. In this research, proposes a compressive sampling image reconstruction based on sparse representation of the image in wavelet transform domain. At the base projections Φ used random orthogonal matrix [18]. As for the reconstruction algorithm compared Orthogonal Matching Pursuit (OMP) and Iteratively Reweighted Least Squares (IRLS) because it is greedy. OM Block diagram compressive sampling is design as shown on figure 3.

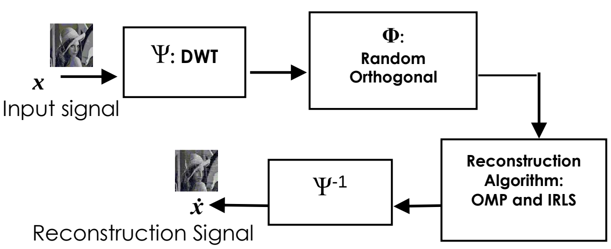


Figure 2: Block Diagram Compressive sampling

The input are images for 8-bit 256x256 pixel, Lena. Comparing parameters PSNR, compression ratio and CPU time at several measurement of M.

4.0 RESULTS AND DISCUSSION

A compression requires the accumulation of energy on the transformation of the region only to a small number of coefficients so that a high compression rate with high quality reconstruction. In the experiment used wavelet transform with the vector base as shown in Figure 3 below:

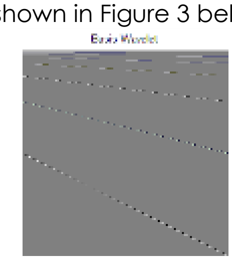


Figure 3: Basis Wavelet

The original image are used Lena with a size of 256x256. Figures 4 and 5 show the original image and reconstructed at measurement coefficient M = 50 , M = 100 , M = 150 and M = 200 on the experimental of OMP and IRLS algorithm.

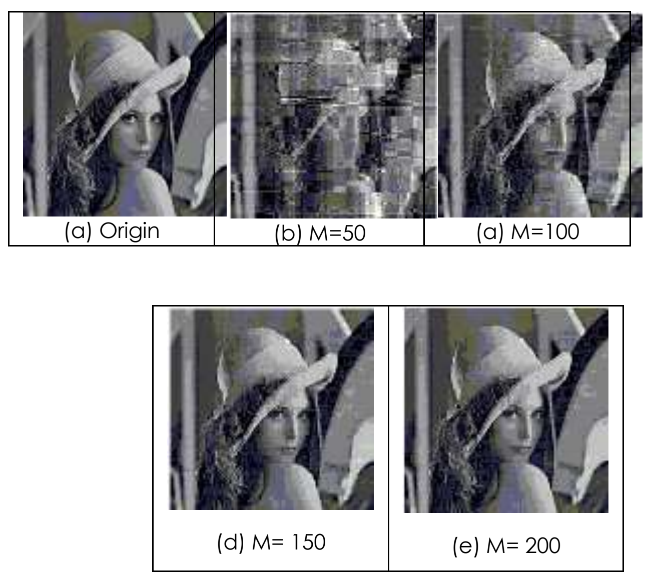
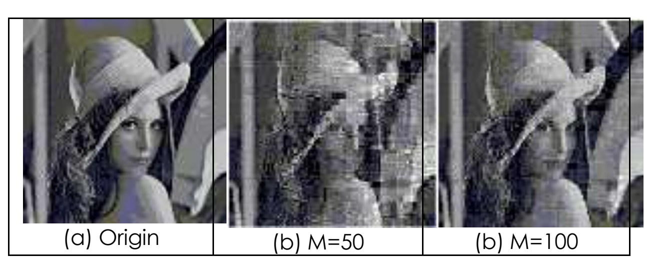


Figure 4: OMP Image Reconstruction



(d) M=150 (e) M=200

Figure 5: IRLS Image Reconstruction

From the results shown above indicate that the larger the coefficient of M, the better the image quality. The picture quality is closer to the original image is shown by the high value of PSNR. The relationship between PSNR with M is shown in Figure 6 below, where PSNR value increased by increasing the number of measurement coefficient M.

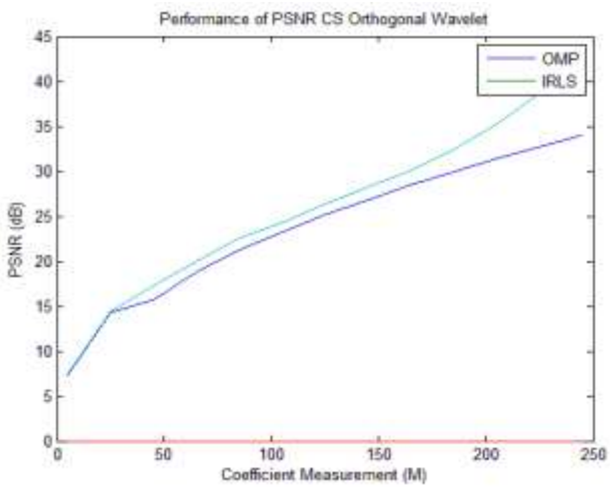


Figure 6: PSNR

Figure 7 shown compression ratio. As the amount of measurement coefficient increases, compression ratio decreases.

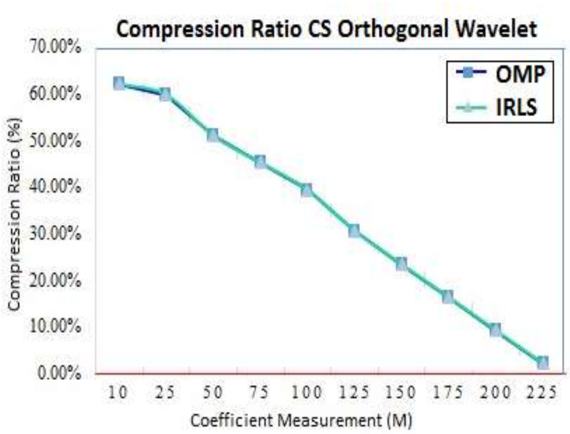


Figure 7: Compression Ratio

Figure 8 shows the relationship between the processing time and the amount of measurement coefficient M. While the measurement coefficient increased, the time required for compressive sampling process increased too.

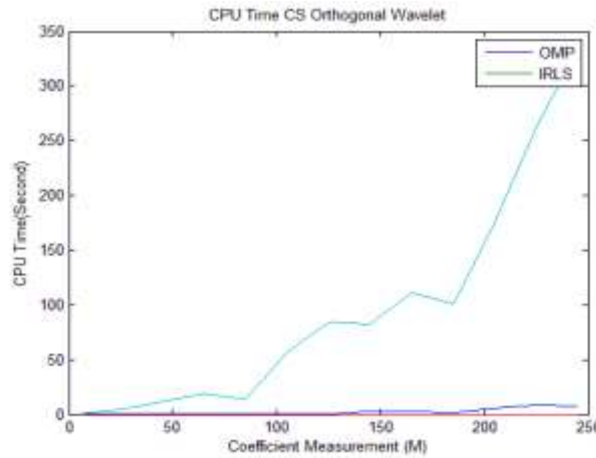


Figure 8: CPU Time

5.0 CONCLUSION

Image reconstruction can obtained only a few number of sample using compressive sampling. So that can reduce bandwidth needed for transmission and reduce the space of storage. The conclusions for this research are as follows:

1. The quality of image reconstruction depends on the number of coefficient measurement. The better quality of image, more measurement M are gotten
2. More measurement M increase CPU time
3. More measurement M decrease compression ratio
4. PSNR and CPU time of IRLS is bigger than OMP. Its means that IRLS given better quality image reconstruction but the process needed longer time.

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